ONS(ONC(SC))98/07

ONE NUMBER CENSUS STEERING COMMITTEE

Weighting versus imputation in the One Number Census

1. This paper describes how an individual level micro database consistent with One Number Census estimates can be created. The imputation and weighting approaches are discussed and the pros and cons of each method highlighted. An illustrative example of weighting and imputation is included

2. An alternative approach to the problem of creating a imputed database with consistent household and individual values will be given in Steering Committee Paper ONS(ONC(SC))98/10, to be distributed at the meeting.

3. The Steering Committee are asked to:

a) note the paper;

b) provide any comments at the meeting on the 27 April 1998, or in writing by 10 May 1998.

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Creating an Individual Level Micro Data Base Consistent with One Number Census Estimates: Imputation vs. Weighting

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Summary

Issues associated with weighted tabulation of "counted" census records rather than unweighted tabulation of an imputed census data base containing synthetic entries for "missing" records are explored in this paper. There is no real gain or loss in choosing between these two approaches if the only aim is to create a set of micro level census tabulations for the distribution of individuals by age, sex and location. The imputation approach becomes slightly more complicated than the weighted approach if other individual-based micro level tabulations are required, involving the use of missing data imputation methodology under development for the 2001 Census. If internally consistent household and individual level tabulations are required, however, than both approaches become considerably more complicated. In particular, the imputation approach seems unworkable in this context, while the weighting approach can be modified via calibration technology. In our opinion, the weighting approach seems to offer the more practical solution to the problem of micro level tabulation in the 2001 Census.

1. Introduction

The current proposal for the One Number Census (ONC) is to use data collected by the Census Coverage Survey (CCS) to estimate the probability of individuals being counted in the census, and then to use these estimated probabilities to impute “missing” individuals within counted households as well as within “missed” households. The aim being to provide a ‘synthetic’ census data base, adjusted for undercoverage, from which all tables can easily be derived. An alternative approach is to use the estimated probabilities to weight the data base of observed census information.

This paper discusses the imputation and weighting approaches and the pros and cons of each method. For completeness it begins with a recap of the methodology for estimating the probability that a particular individual gets counted in the census, as this is the common starting point for both methods.

2. Modelling individual census undercount using the CCS

Let ij index an individual i within a household j “counted” by the CCS in location klm (postcode k within enumeration district l of county/district/group/area m). On the basis of information collected in the CCS, this individual/household combination can be matched with those individuals/households providing census returns for the same location.

In theory, each household in location klm can be counted/not counted by the census/ CCS (4 possible outcomes) and each individual can also be counted/not counted by the census/CCS (4 possible outcomes), leading to 16 possible outcomes for an individual/household combination in this matching process. However, we assume that only five of these are observable:

(a) both individual (i) and household (j) are counted in the census and the CCS;
(b) household (j) is counted in the census and the CCS, individual (i) is counted in the census, but not the CCS;
(c) household (j) is counted in the census and the CCS, individual (i) is counted in the CCS, but not the census;
(d) household (j) is counted in the census, but not the CCS;
(e) household (j) is counted in the CCS, but not the census.

Diagramatically we have the following situation, with “X” denoting a cell where a count is “impossible”.

	$i \in$ CCS $j \in$ CCS	$i \notin$ CCS $j \in$ CCS	$j \notin$ CCS
$i \in$ Census $j \in$ Census	(a)	(b)	(d)
$i \notin$ Census $j \in$ Census	(c)	X	X
$j \notin$ Census	(e)	X	X

This model assumes that all individuals within unmatched households found by the CCS are enumerated by the CCS and missed by the census, and conversely all individuals within unmatched households found by the census are enumerated by the census and missed by the CCS. Furthermore, it assumes that for a household counted in both the census and the CCS, no individuals are missed by both CCS and census, and all households in a location are counted either in the census or the CCS or both.

Note that the above scenario is an extension of the one described in the ONC consultation document.

Let π_{sijklm} denote the probability of outcome s above, s = a, b, c, d, e. Assuming "perfect matching", the sum of these probabilities is then one, since every household/ individual combination must fall into one and only one of these categories. Let X_{ijklm} denote the vector of individual characteristics for individual i in household j, and let Z_{jklm} denote the vector of household characteristics of household j. Note that components of Z_{jklm} will typically depend on the characteristics of ALL individuals in household j. Furthermore, it is possible that both census and CCS "values" for both X_{ijklm} and Z_{jklm} will be available (e.g. cases a, b, c above). In such situations we will take the "Census value" as the defining value.

The multinomial logistic model proposed for these probabilities is then given by

$$\log\left(\frac{\pi_{sijklm}}{\pi_{aijklm}}\right) = \alpha_{sm} + \beta'_{sm}X_{ijklm} + \gamma'_{sm}Z_{jklm} + \xi_{sklm}$$

where α_{sm}, β_{sm} and γ_{sm}, s = b, c, d, e are unknown values common to all individuals within area m and the ξ_{sklm} are correlated random effects specific to location klm.

Given the outcome of the matching process described above, this model can be fitted to the combined census/CCS data for the sampled locations, and estimates (denoted by "hats" below) obtained for α_{sm}, β_{sm} and γ_{sm}, together with optimal predictions of the values of the ξ_{sklm}.

3. Predicting individual undercount propensities for all census respondents

Since the CCS sample only covers a small proportion of the locations covered by the census, the next step is to use these estimates to assign a set of "fitted" probabilities to all individuals counted in the census. Suppose individual I within household J was counted by the census at location KLM. Then the probability that this event occurred can be estimated by

$$\hat{\pi}_{IJKLM} = \frac{1 + \exp(\hat{L}_{bIJKLM}) + \exp(\hat{L}_{dIJKLM})}{1 + \sum_{s \neq a} \exp(\hat{L}_{sIJKLM})}$$

where

$$\hat{L}_{sIJKLM} = \hat{\alpha}_{sM} + \hat{\beta}'_{sM}X_{IJKLM} + \hat{\gamma}'_{sM}Z_{JKLM} + \hat{\xi}_{sKLM}$$

and the $\hat{\xi}_{sKLM}$ are defined by averaging the corresponding predictions from the h "closest" CCS locations to location KLM. At present a value of h = 4 is proposed, but further work is necessary to establish a good "bandwidth" for this averaging process.

It is important to note that the "closeness" metric that will be used in the above computation will NOT be purely spatial. It will also allow for other types of similarities between locations (e.g. hard to count index value). Note also that this approach assumes that all individuals counted in the census will have values for X and Z.

An estimate of the total number of individuals in age-sex category A who would be counted by both the census and the CCS applied to ALL census locations in area M is then

$$\tilde{T}_{AM} = \sum_{KL \in M} \sum_{IJ \in KLM} \frac{\Delta(I \in A)}{\hat{\pi}_{IJKLM}}$$

where Δ denotes an indicator function which takes the value 1 if its argument is true, and is zero otherwise. Typically, $\tilde{T}_{AM}$ will differ from the One Number Census estimate $\hat{T}_{AM}$ for the same age-sex category. It is proposed that the latter estimate be based on a combination of regression and dual system estimation (DSE) methodology (plus national level demographics) applied to the census/CCS data, adjusting for both census and CCS undercoverage.

4. Creating "coverage weights" for all individuals counted in the census

The next step is to create a "weight" to go with each individual counted by the census. This weight is obtained by calibrating the inverse probability weights defined by the multinomial logistic model described above so that they recover the One Number Census age-sex estimates $\hat{T}_{AM}$. That is, for "census counted" individual I in age-sex category A in household J in location KLM, we define a "coverage weight"

$$w_{IJKLM} = \left(\frac{1}{\hat{\pi}_{IJKLM}} \right) \left(\frac{\hat{T}_{AM}}{\tilde{T}_{AM}} \right).$$

This weight will typically be greater than one. However there is no guarantee of this. The total number of individuals in age-sex category A "missed" by the census in location KLM is then

$$\hat{M}_{AKLM} = \sum_{IJ \in KLM} (w_{IJKLM} - 1) \Delta(I \in A).$$

Note that the above steps ensure that "coverage weighted" age-sex tabulations obtained from the census database coincide with the age-sex totals generated by the CCS at level M of the location index (typically this will be Local Authority District level). However, the methodology can be easily extended to ensure that these weights are also "calibrated" with respect to other variables (not just age and sex) for which "reliable" CCS/demographic estimates are available at some agreed level (this may be more aggregated than LAD level).

5. Imputation vs. weighting as a methodology for micro level undercount adjustment

At this stage we have derived the coverage weights for individuals in age-sex category A in location KLM. How do we now proceed in order to produce the required output? As things stand there are two alternative approaches that can be taken - weighting or imputation.

At present, it is proposed that the CCS-based estimated number of people "missed" by the census be used to impute these people in the census data base. That is, we "remove" the undercount by generating $\hat{M}_{AKLM}$ synthetic age-sex category A person records in the census data base for location KLM. This is similar to the approach allegedly taken by the US Bureau of the Census in their 1990 undercount adjustment, where a hotdeck methodology was used to carry out this "whole person" imputation. So far we have been unable to confirm the veracity of this (investigations are still proceeding). A potential hotdeck methodology for this imputation is described below. However, as will become obvious, a basic problem with this approach is the issue of creating or adjusting existing households in which to put the imputed "people".

An alternative is to not impute, but to create a weighted census data base, using the weights w_{IJKLM} above. All tabulations derived from this data base would be weighted tabulations. At the person level such tabulation is a straightforward exercise given modern computing facilities - all that is required is the creation of an extra "coverage weight" field in the census data base.

6. A hotdeck imputation method

The basic assumption underlying this imputation method is the existence of D "Impute Classes", within which individual coverage probabilities are essentially the same (or at least do not vary very much) and which reflect individual and household differences as far as coverage probabilities are concerned.

Starting with a randomly chosen location in a "processing block" (typically an enumeration district or a collection of enumeration districts), and processing the locations in this "block" in random order, the idea is to cumulate both the frequency of individuals in each Impute Class as well as their coverage weights. The cumulated frequencies for each Impute Class is referred to as its "imputed cumulant" sequence, while the cumulated weights for this class is referred to as its "weighted cumulant" sequence. Since coverage weights are typically greater than one, these two cumulated sequences eventually move apart. When the weighted cumulant sequence is 0.5 or more greater than the imputed cumulant sequence, an "extra" individual with coverage weight zero in the Impute Class is created in the location where this happens. This results in the imputed cumulant sequence moving ahead of the weighted cumulant sequence, and the process starts over.

Eventually all individuals in the location are "counted". The difference between the starting and finishing values of the imputed cumulant sequences for the different Impute Classes then defines the imputation-based counts of individuals in these classes in that location. By construction, these imputation-based counts will all be within 0.5 of the weighted counts for that location.

The next location is then processed, starting from the cumulants generated by the location just completed. This process continues until the weighted cumulant sequence for (typically) another Impute Class gets too far ahead of its imputed cumulant sequence, and another "pseudo individual" in that Class is created. Eventually all individuals in the processing block are "counted" in this way.

Tables 1 to 3 illustrate this process in the simple case where there are D = 7 Impute Classes, defined by

- Class 1 = children under 5
- Class 2 = children between 5 and 15
- Class 3 = females between 16 and 25
- Class 4 = males between 16 and 25
- Class 5 = females between 26 and 59
- Class 6 = males between 26 and 59
- Class 7 = people 60 and over.

Here the location corresponds to a postcode and the "processing block" has been (arbitrarily) set at 40 postcodes with similar census characteristics (e.g. all from the same enumeration district).

Several issues arise given such a hotdeck methodology is adopted. These are

1. Definition of the Impute Classes;
2. Definition of the processing block;
3. Choice of "starting" location;
4. Ordering of locations in the processing sequence;
5. Definition of the "census characteristics" of an imputed individual.

Of these, probably the most contentious will be issue (5). The hotdeck methodology merely puts people into Impute Classes. This will typically be insufficient to "define" an imputed person as a complete census record. One possibility will be to "replicate" the last "counted" individual before the imputee and place him/her in that individual's household. However this has the drawback of artificially distorting the structure of that household. It will also lead to problems with census edit checks (e.g. a wife with two husbands, one real, the other imputed). Another possibility is to give the imputee an "average" set of characteristics for persons falling in the Impute Class. Again, this can lead to the creation of nonsensical person records. Ideally, what one would want to do is to "give" an imputee the characteristics of the last "real" individual processed before his/her creation, but then place him/her in a household where such a "placement" makes sense. Exactly how this can be accomplished in a reasonably automatic way remains to be seen.

These problems do not rise, of course, with the weighted approach. However, this is not to say that weighting does not also lead to problems. Essentially, the major problem with weighting is treatment of roundoff error in tabulations. It seems reasonable to adopt the policy of rounding to the nearest integer in weighted census tabulations. However, the level at which such rounding occurs can have a major impact. There is no guarantee that rounding at location level and then adding across locations will have the same end result as (weighted) adding across locations and then rounding. If weighted tabulation is adopted for the 2001 Census, it seems sensible to round at the most "disaggregated" level of census tabulation (ED?) that is publicly released.

7. Dealing with household tabulations

A problem with both the hotdeck and weighting approaches discussed above is that they are essentially based on individual level coverage weights. That is, the weights w_{IJKLM} are individual level weights, even though they take account of household structure. This can lead to inconsistency problems in the derivation of weighted household tabulations from the census. It also means that the number of households contributing to census tabulations will be less than the actual number of households.

A resolution of this problem can be achieved by computing "census coverage" weights for census counted households in addition to the individual level "census coverage" weights described above. In particular, the steps outlined in sections 2 - 4 above can be applied to CCS/census data on households, rather than individuals, to develop such a weighting process. As before, let j denote a household in census/CCS location klm. Then j can be classified as follows:

(1) counted in both the census and the CCS;
(2) counted in the census, but missed by the CCS;
(3) counted by the CCS, but missed by the census.

Again, based on the information obtained by matching the census/CCS data for location klm, we can fit a multinomial logistic model to the probabilities associated with household j being in one of these states. Let θ_{tjklm} denote the probability of outcome t, t = 1, 2, 3. Then

$$\log\left(\frac{\theta_{tjklm}}{\theta_{1jklm}}\right) = \delta_{tm} + \lambda'_{tm} Z_{jklm} + \eta_{tklm}$$

where now δ_{tm} and λ_{tm}, t = 2, 3, define the fixed effects in the model, and the η_{tklm} are the correlated random effects specific to location klm.

The same fitting + extrapolation process (including "spatial" smoothing as a method for extrapolating random effects) used in the individual coverage situation can then be applied to the above model to obtain the estimated probability that household J in location KLM is counted by the census:

$$\hat{\theta}_{JKLM} = \frac{1 + \exp(\hat{Q}_{2JKLM})}{1 + \sum_{t \neq 1} \exp(\hat{Q}_{tJKLM})}$$

where

$$\hat{Q}_{tJKLM} = \hat{\delta}_{tM} + \hat{\lambda}'_{tM} Z_{JKLM} + \hat{\eta}_{tKLM}.$$

The "census coverage" weight associated with household J is then just the inverse of the fitted probability $\hat{\theta}_{JKLM}$ above.

Since in principle CCS-based estimates of the total number of households of different "types" can be computed, the same "calibration" process that adjusts individual level census coverage weights so they recover ONC estimates can be applied to these household level census coverage weights. All household tabulations derived from the census would then be based on these calibrated census coverage weights.

8. Consistency between individual and household level weights

A remaining problem with this household weight vs. individual weight approach is maintaining consistency between the two sets of weights. To illustrate, suppose a tabulation is required for the total number of individuals in area M classified by some characteristic with levels indexed by c = 1, 2, ..., C. An individual entry in this tabulation is then a count T_{cM} of the number of individuals in this area with characteristic level = c. Using the individual level census data base, this count can be estimated by

$$\hat{T}_{cM}^{ind} = \sum_{KL \in M} \sum_{IJ \in KLM} w_{IJKLM} \Delta(I \in c)$$

where w_{IJKLM} is the individual census coverage weight for individual I in household J in location KLM, and Δ, as usual, is the indicator function which takes the value 1 if its argument is true and is zero otherwise. However, if household affiliation is available on this data base, and the household census coverage weight w_{JKLM} for individual I in household J is also provided, then an alternative estimate of T_{cM} is

$$\hat{T}_{cM}^{h/h} = \sum_{KL \in M} \sum_{J \in KL} w_{JKLM} \sum_{I \in J} \Delta(I \in c)\ .$$

There is no guarantee that $\hat{T}_{cM}^{ind}$ and $\hat{T}_{cM}^{h/h}$ will be the same, and, in general, it is theoretically possible that there may be a considerable difference between these values.

This problem of consistency between household and individual weights is well known in the sample survey literature, and there are methods for modifying household weights so that consistency with individual level weighting is assured for selected individual level characteristics. Essentially, this amounts to "calibrating" the household weights so that household-weighted counts for these characteristics are equal to corresponding individual-weighted counts. This issue will need to be addressed if a weighting approach to micro-level census tabulation is adopted.

A further issue arises if an imputation strategy is adopted and household as well as individual coverage weights are calculated. Clearly, the hotdeck methodology for imputing individuals can easily be modified for imputing households. However, it is not at all clear how the two imputation processes (imputation of individuals and imputation of households) can be integrated. In a postcode where both an individual as well as a household have been imputed, does one place the imputed individual in the imputed household? This may make sense if the imputed individual is an adult. But it is nonsensical if the imputed individual is a child. In the latter case, one then has a postcode with a "ghost" imputed household containing no individuals. One way of getting around this would be to "synchronise" household and individual imputation so that a household is imputed only in a location where there are sufficient "imputees" to be placed in it. In turn this will probably require "rolling forward" individual imputations. That is, we only impute individuals after a "critical imbalance" between the weighted and imputed "all persons" counts are reached, and not just after this imbalance is reached within a particular Impute Class (as suggested in section 6). Exactly what this "critical imbalance" value should be, and how one then handles imputations within "counted" households remains an area where further research will need to be done.

9. Weighting verses imputation: A user perspective

If adjustment, either by weighting or imputation, is restricted to individual level variables for which the undercount has been measured i.e. age, sex and area, and this is the limit of our ambition for the ONC, then it is likely that tabular output from either imputation or weighting will be the same. That is, the methodology will be transparent to the majority of Census users who require data in the form of tables. However there are a minority of users who require direct access to the data to carry out their own analysis. This demand has traditionally been met through the provision of samples of anonymised data (SARs). We would expect these users to favour weighting over imputation as it preserves the census data as collected and we will be able to provide them with (quite straightforward) advice on how to use weights in tabulation. Here weighting has the advantage of making the census quite "open". Users of micro data can immediately "see" where the census had coverage problems, and they also have the option of weighting or not weighting, but must weight if they want to be consistent with ONC figures.

However a general issue arises with either method (but particularly with imputation) if we consider extension beyond age, sex and area adjustment. Suppose we overcome the problems listed earlier and successfully develop an imputation methodology to put synthetic people into real or synthetic households. These people will have age and sex values that accord with our best estimate of the distribution of the amount of under enumeration in the population. Given this, it is straightforward to use the existing missing item imputation system to fill in their other variables (activity last week, religion etc.) and thus have a completely filled in data base. However in doing this we do not have control of the resulting distributions for these variables and distortions may arise. We have the option of not filling in these "related" variables but then tables will not add up.

This issue is not restricted to imputation. If we apply the weighting procedure developed in sections 2 - 4 to all variables in the census data base, then, at some level of aggregation, the weighted distribution of variables other than age and sex will not generally be the same as, for example, the same distributions estimated from demographic sources or the CCS. However, with weighting we do have the option of imposing extra calibration constraints to ensure this agreement. Unfortunately, if there are too many such constraints, there is no guarantee that a suitable set of weights will be found, so one must be cautious in introducing too many "external" constraints. In general, the advice is to only impose those constraints which are based on variables that are "related" to the key age and sex variables of interest in the census.

10. Summary

IMPUTATION:

For

a) Tabulation straightforward, everything “adds up”. Whole people in tables.
b) Can use the Donor Imputation methods currently being developed to “fill in” values of “missing” variables for “imputees.
c) Methodology transparent to users of tabular data.

Against

a) Hotdeck methodology untested, anecdotal evidence of whole person imputation in USA.
b) A nontrivial extension is required to create synthetic households.
c) Presentation: A synthetic rather than real data base.

WEIGHTING

For

a) Standard methodology exists - and extension to household tabulation is conceptually straightforward.
b) Users of micro data expected to prefer this approach.
c) Can theoretically calibrate against other variables.
d) Presentation : We have maintained the integrity of the data we collected and have not created artificial (statistical) people.

Against

a) Some adjustment of previous census tabulation methods required.
b) Would have to develop methods for dealing with households - but solutions exist.
c) Policy on rounding in tables required.

Table 1: An illustrative depiction of census data collected for one postcode, with individual coverage weights attached. Note that for the purpose of showing how the weighting and imputation approaches work, "Impute Class" here is defined solely in gender of gender and age. In practice, "Impute Class" will also reflect other individual as well as household characteristics.

HH	HH/Type	HH/Size	Gender	Age	Weight	Impute Class
1	1	1	F	23	1.03	3
2	2	4	F	39	1.00	5
			M	45	1.00	6
			F	14	1.00	2
			F	8	1.00	2
3	2	2	F	24	1.05	3
			M	24	1.05	4
4	1	1	M	36	1.10	6
5	5	1	F	58	1.00	7
6	2	3	M	39	1.00	6
			F	38	1.00	5
			M	12	1.00	2
7	4	6	M	63	1.05	7
			F	36	1.00	5
			M	38	1.00	6
			M	19	1.02	4
			M	17	1.02	4
			F	8	1.00	2
8	6	2	M	68	1.01	7
			F	59	1.01	7
9	3	3	F	28	1.03	5
			M	8	1.02	2
			M	6	1.02	2
10	2	3	F	39	1.00	5
			M	37	1.00	6
			F	15	1.00	2
11	5	1	F	63	1.00	7
12	3	3	M	26	1.01	6
			F	24	1.01	3
			F	1	1.10	1

Table 2: This shows the data processing for the illustrative postcode. The first row is the “carryover” in cumulated counts from the previous postcode processed, while the last row is the same “carryover” to the next postcode to be processed. Note that individuals contribute their weight to the cumulated “wtd” total for their Impute Class, but only their frequency (= 1) to the cumulated “imputd” total for this class, until the cumulated “wtd” total is greater than the cumulated “imputd” total by 0.5 or more, in which case an “extra’ (i.e. imputed) individual is added to the cumulated “imputd” total. In the example below, one individual has been imputed for this postcode, in Impute Class 4.

Impute Class 1 Cumulated		Impute Class 2 Cumulated		Impute Class 3 Cumulated		Impute Class 4 Cumulated		Impute Class 5 Cumulated		Impute Class 6 Cumulated		Impute Class 7 Cumulated	
imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd
10	**9.90**	**25**	**25.00**	**18**	**18.31**	**19**	**19.42**	**24**	**24.00**	**27**	**27.03**	**16**	**15.85**
11	11.00	26	26.00	19	19.34	20	20.47	25	25.00	28	28.03	17	16.85
		27	27.00	20	20.39	21	21.49	26	26.00	29	29.13	18	17.90
		28	28.00	21	21.40	22	22.51	27	27.00	30	30.13	19	18.91
		29	29.00			**23 (I)**		28	28.03	31	31.13	20	19.92
		30	30.02					29	29.03	32	32.13	21	20.92
		31	31.04							33	33.14		
		32	32.04										
11	**11.00**	**32**	**32.04**	**21**	**21.40**	**23**	**22.51**	**29**	**29.03**	**33**	**33.14**	**21**	**20.92**

Table 3: Overview of the cumulated imputed and weighted counts in the 40 postcodes making up a hypothetical processing block (e.g. an ED). The actual imputed count and weighted count for each cell (postcode by Impute Class) are shown in parentheses below their respective cumulated counts. The illustrative postcode shown in Tables 1 and 2 is postcode 4 below. Note that the method of imputing will always result in a total weighted count for an Impute Class which is within 0.5 of the corresponding total imputed count. However, imputed counts in cells in which a person has been imputed can be up to 0.99 greater than the corresponding weighted count for the cell. Cells where an imputed person has been "created" are shown in bold below.

PCode	Impute Class 1 Cumulated		Impute Class 2 Cumulated		Impute Class 3 Cumulated		Impute Class 4 Cumulated		Impute Class 5 Cumulated		Impute Class 6 Cumulated		Impute Class 7 Cumulated	
	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd	imputd	wtd
1	0	0	6	6.00	4	4.04	7	7.16	8	8.00	10	10.00	5	5.20
2	**6**	**5.50**	15	15.00	12	12.26	13	13.25	17	17.00	19	19.01	11	11.42
	(6)	**(5.50)**	(9)	(9.00)	(8)	(8.22)	(6)	(6.09)	(9)	(9.00)	(9)	(9.01)	(6)	(6.22)
3	10	9.90	25	25.00	18	18.31	19	19.42	24	24.00	27	27.03	**16**	**15.85**
	(4)	(4.40)	(10)	(10.00)	(6)	(6.05)	(6)	(6.17)	(7)	(7.00)	(8)	(8.02)	**(5)**	**(4.43)**
4	11	11.00	32	32.04	21	21.40	**23**	**22.51**	29	29.03	33	33.14	21	20.92
	(1)	(1.10)	(7)	(7.04)	(3)	(3.09)	**(4)**	**(3.09)**	(5)	(5.03)	(6)	(6.11)	(5)	(5.07)
5	12	12.10	39	39.12	**28**	**27.68(**	28	27.72	37	37.03	38	38.15	23	22.98
	(1)	(1.10)	(7)	(7.08)	**(7)**	**6.28)**	(5)	(5.21)	(8)	(8.00)	(5)	(5.01)	(2)	(2.06)
6	14	14.30	49	49.15	34	33.72	32	32.02	46	46.05	46	46.25	27	27.03
	(2)	(2.20)	(10)	(10.03)	(6)	(6.04)	(4)	(4.30)	(9)	(9.02)	(8)	(8.10)	(4)	4.05)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
40	57	56.90	268	268.02	189	189.36	201	200.84	357	357.15	368	368.18	183	183.35
	(1)	(1.10)	(7)	(7.01)	(8)	(8.12)	(7)	(7.08)	(8)	(8.01)	(7)	(7.02)	(3)	(3.15)